Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa., and HON. JOSIAH H. DRUMMOND, LL. D., Portland, Me.

Let $x+y+z=s$. Then (1)+(2)+(3) gives $xy+xz+yz=\frac{1}{2}(a+b+c)s$..(4).
(4)−(1) gives $yz=\frac{1}{2}(b+c-a)s$....(5).
(4)−(2) gives $xz=\frac{1}{2}(a+c-b)s$....(6).
(4)−(3) gives $xy=\frac{1}{2}(a+b-c)s$....(7).
(6)÷(7) gives $z/y=(a+c-b)/(a+b-c)$....(8).

(8) multiplied by (5) gives $$z=\sqrt{\frac{(a+c-b)(b+c-a)s}{2(a+b-c)}}\ldots\text{(9)}.$$

Similarly, $$y=\sqrt{\frac{(a+b-c)(b+c-a)s}{2(a+c-b)}},\quad x=\sqrt{\frac{(a+b-c)(a+c-b)s}{2(b+c-a)}}\ldots\text{(10, 11)}.$$

(9)+(10)+(11) gives $$\sqrt{s}=\frac{2ab+2ac+2bc-a^2-b^2-c^2}{\sqrt{[2(a+b-c)(a+c-b)(b+c-a)]}}\ldots\text{(12)}.$$

(12) in (9), (10), (11) gives

$$x(b+c-a)=y(a+c-b)=z(a+b-z)=\tfrac{1}{2}(2ab+2ac+2bc-a^2-b^2-c^2).$$

Also solved by *J. M. BOORMAN*, *W. H. CARTER*, *C. C. CROSS*, *LESLIE L. LOCKE*, *COOPER D. SCHMITT*, *ELMER SCHUYLER*, *J. SCHEFFER*, *B. F. YANNEY*, *J. W. YOUNG*, and *M. A. GRUBER*.

GEOMETRY.

138. Proposed by JOHN M. HOWIE, Professor of Mathematics, The Nebraska State Normal, Peru, Neb.

K is the middle point of any chord AB of a given circle. CD and EF are any two chords passing through K. CF and ED intersect AB at M and N, respectively. Prove that KM equals KN.

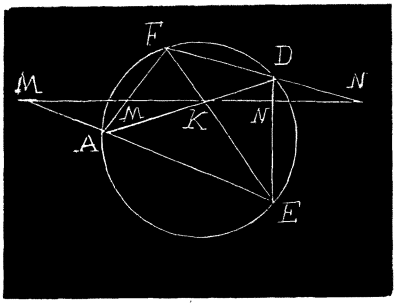
I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.; P. S. BERG, B. Sc., Larimore, N. D.; and F. B. FILLMAN, Chester, Pa.

CASE I. N and M within the circle.
$\triangle DKN/\triangle CKM=DK.NK/CK.MK$....(1).
$\triangle EKN/\triangle FKM=EK.NK/FK.MK$....(2).
$\triangle CKM/\triangle EKN=CM.CK/EN.EK$....(3).
Multiplying (1) by (3),

$$\frac{\triangle DKN}{\triangle EKN}=\frac{DK.NK.CM}{MK.EN.EK}=\frac{DN}{EN}\ldots\text{(4)}.$$

Multiplying (2) by (3),

$$\frac{\triangle CKM}{\triangle FKM}=\frac{NK.CM.CK}{EN.FK.MK}=\frac{CM}{FM}\ldots\text{(5)}.$$



From (4) and (5),
$NK/MK=DN.EK/DK.CM\ldots.(6)$.
$NK/MK=EN.FK/CK.FM\ldots.(7)$.
Multiplying (6) by (7),

$$\therefore\frac{NK^2}{MK^2}=\frac{DN.EN.EK.FK}{CM.FM.DK.CK}\ldots.(8).$$

But $EK.FK=DK.CK=AK^2$.

$$DN.EN=BN.AN=(BK-KN)(BK+KN)=BK^2-KN^2.$$

$$CM.FM=AM.MB=(BK-MK)(BK+MK)=BK^2-KM^2.$$

$$\therefore\frac{NK^2}{MK^2}=\frac{BK^2-NK^2}{BK^2-MK^2}.$$

$\therefore\ NK^2.BK^2-NK^2.MK^2=MK^2.BK^2-NK^2.MK^2$.
$\therefore\ NK^2.BK^2=MK^2.BK^2$.
$\therefore\ NK=MK$.

Case II. N and M without the circle.
$\triangle DKN/\triangle CKM=DK.NK/CK.MK\ldots.(9)$.
$\triangle FKN/\triangle EKM=FK.NK/EK.MK\ldots.(10)$.

$\angle KDN$ is the supplement of $\angle FDK$ and therefore the supplement of $\angle KEC$.

$\therefore\ \triangle EKM/\triangle DKN=KE.ME/KD.ND\ldots.(11)$.
Multiplying (9) by (11),

$$\frac{\triangle EKM}{\triangle CKM}=\frac{NK.KE.ME}{ND.CK.MK}=\frac{ME}{MC}\ldots.(12).$$

Multiplying (10) by (11),

$$\frac{\triangle FKN}{\triangle DKN}=\frac{FK.NK\ ME}{KD.ND.MK}=\frac{NF}{ND}\ldots.(13).$$

From (12) and (13),
$NK/MK=ND.CK/MC.KE\ldots.(14)$.
$NK/MK=NF.KD/FK.ME\ldots.(15)$.
Multiplying (14) by (15),

$$\frac{NK^2}{MK^2}=\frac{ND.NF.CK.KD}{MC.ME.FK.KE}=\frac{ND.NF}{MC:ME}=\frac{NB.NA}{MB.MA}=\frac{(NK-BK)(NK+BK)}{(MK-BK)(MK+BK)}$$

$$=\frac{NK^2-BK^2}{MK^2-BK^2}.$$

$\therefore NK^2.MK^2-NK^2.BK^2=NK^2.MK^2-MK^2.BK^2$.

$\therefore NK^2.BK^2=MK^2.BK^2$.

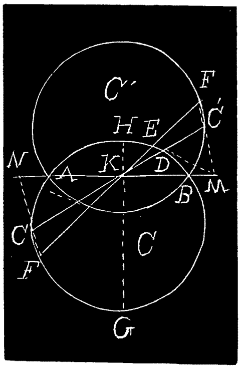
$\therefore NK=MK$.

II. **Solution by F. E. MILLER, A. M., Professor of Mathematics, Otterbein University, Westville, Ohio, and P. C. CULLEN, Indianola, Neb.**

K the mid-point of chord AB, and CD and EF chords through K.

To prove that the joins CF and ED meet AB equidistant from K.

Through A and B draw circles $C'\equiv C$ and produce CD and EF to C' and F'.



From symmetry we see that FC and $F'C'$ meet AB equidistant from K and are parallel.

$\angle EDC=\angle EFC=\angle EF'C'$ and hence $EDC'F'$ are concyclic. Then is $F'C'$ and ED meet in M, $MD.ME==MC'.MF'$, or the tangents from M to the circles C and C' are equal and therefore M is on the radical axis, *i. e.* on AB.

$\therefore NK=MK$.

Again by projection.

Project circle C on a plane through AB so that the projection of HG perpendicular to AB may have the projection of K as its mid-point. Then the circle becomes an ellipse with K as center and CD and EF as diameters, and ED and FC meeting the major axis AB equally distant from the center K. But points on AB are not changed by the projection. Therefore N and M are always equidistant from K.

A second solution by Analytical Geometry was furnished by Professor Zerr.

CALCULUS.

104. **Proposed by M. E. GRABER, Heidelberg University, Tiffin, Ohio.**

Find the differential equation corresponding to $\sqrt{(1-x^2)}+\sqrt{(1-y^2)}=[a(x-y)]$.

Solution by G. B. M. ZERR, A. M., Ph. D., The Temple College. Philadelphia, Pa.; COOPER D. SCHMITT, A. M., University of Tennessee, Knoxville, Tenn.; and the PROPOSER.

$\sqrt{(1-x^2)}+\sqrt{(1-y^2)}=a(x-y)\ldots.(1)$.

$xdx/\sqrt{(1-x^2)}+ydy/\sqrt{(1-y^2)}=a(dy-dx)\ldots.(2)$.

Eliminating a between (1) and (2) and reducing we get

$$\{xy-1-\sqrt{[(1-x^2)(1-y^2)]}\{\sqrt{[1-y^2]}dx$$
$$=\{xy-1-\sqrt{[(1-x^2)(1-y^2)]}\}\sqrt{[1-x^2]}dy.$$

$\therefore dx/dy=\sqrt{(1-x^2)}/\sqrt{(1-y^2)}$.